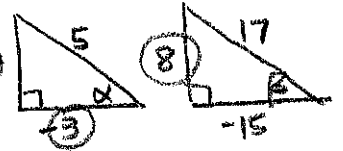


GSE PreCalculus
Test 5B Review: Trig Identities

Name Key

Given that α and β are in quadrant 2 and $\sin \alpha = \frac{4}{5}$ and $\cos \beta = -\frac{15}{17}$, find:



1. $\cos \alpha = -3/5$

2. $\sin \beta = 8/17$

3. $\sin(2\alpha) = 2\sin\alpha\cos\alpha = 2(4/5)(-3/5) = -24/25$

4. $\cos(2\beta) = \cos^2\beta - \sin^2\beta = (-15/17)^2 - (8/17)^2 = \frac{161}{289}$

5. $\tan(2\beta) = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{2(-9/15)}{1-(-9/15)^2} = \frac{-18/15}{161/225} = -14/15 \cdot \frac{225}{161} = -240/161$

6. $\cos(\alpha - \beta)$

Use half angle formulas to solve the following

$\cos 157.5^\circ = \cos \frac{315^\circ}{2} = -\sqrt{\frac{1+\cos 315^\circ}{2}} = -\sqrt{\frac{1+\sqrt{2}/2}{2}}$
 $\frac{2+\sqrt{2}}{2} = \frac{2+\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{2+\sqrt{2}}{4}$

8. $\sin 15^\circ = \sin \frac{30^\circ}{2} = \sqrt{\frac{1-\cos 30^\circ}{2}} = \sqrt{\frac{1-\sqrt{3}/2}{2}}$
 $\frac{2-\sqrt{3}}{2} = \frac{2-\sqrt{3}}{2} \cdot \frac{1}{2} = \frac{2-\sqrt{3}}{4}$

Solve.

9. $2\sin^2 x = 2 + \cos x$
 $2(1-\cos^2 x) = 2 + \cos x$
 $2 - 2\cos^2 x - \cos x - 2 = 0$
 $\cos^2 x(-2\cos x - 1) = 0$
 $\cos^2 x = 0 \quad -2\cos x = 1$
 $\cos x = 0 \quad \cos x = -1/2$
 $90^\circ \quad 120^\circ$

10. $2\sin \alpha \cos \alpha = \sin \alpha$
 $2\sin \alpha \cos \alpha - \sin \alpha = 0$
 $\sin \alpha (2\cos \alpha - 1) = 0$
 $\sin \alpha = 0 \quad 2\cos \alpha - 1 = 0$
 $\cos \alpha = 1/2$
 $0^\circ \quad 60^\circ$

11. $\sin^2 x - 3\cos x = 3$
 $1 - \cos^2 x - 3\cos x - 3 = 0$
 $-\cos^2 x - 3\cos x - 2 = 0$
 $-1(\cos^2 x + 3\cos x + 2) = 0$
 $-1(\cos x + 2)(\cos x + 1) = 0$
 $\cos x = -2 \quad \cos x = -1$
 180°

12. $2\sin^2 x = 9\sin x + 5$
 $2\sin^2 x - 9\sin x - 5 = 0$
 $(2\sin x + 1)(\sin x - 5) = 0$
 $2\sin x + 1 = 0 \quad \sin x = 5$
 $\sin x = -1/2$
 -30°

13. $\sin^2 \beta - \sin \beta = 0$
 $\sin \beta (\sin \beta - 1) = 0$
 $\sin \beta = 0 \quad \sin \beta = 1$
 $0^\circ \quad 90^\circ$

Verify the following.

14. $\sin(x+y) + \sin(x-y) = 2\sin x \cos y$

$\sin x \cos y + \cos x \sin y + \sin x \cos y - \cos x \sin y = 2\sin x \cos y$
 $2\sin x \cos y = 2\sin x \cos y$

16. $\sec^4 x - \tan^4 x = 1 + 2\tan^2 x$

$(\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x) = 1 + 2\tan^2 x$
 $(1 + \tan^2 x)(1 + \tan^2 x) = 1 + 2\tan^2 x$
 $(1)(1 + 2\tan^2 x) = 1 + 2\tan^2 x$

15. $\frac{\sin x}{\sin x - \cos x} = \frac{1}{\cos x - \cot x}$

$\sin x(1 - \cot x) = \sin x - \cos x$
 $\sin x - \cos x = \sin x - \cos x$

16. $\cos^2 x(1 + \tan^2 x) = 1$

$\cos^2 x(\sec^2 x) = 1$
 $1 = 1$

17. $\csc 2\theta = \frac{\csc \theta}{2\cos \theta}$

$\frac{1}{\sin 2\theta} = \frac{1}{\sin \theta} \cdot \frac{1}{2\cos \theta}$
 $\frac{1}{2\sin \theta \cos \theta} = \frac{1}{2\sin \theta \cos \theta}$

18. $\sec 2\theta = \frac{\sec^2 \theta}{2 - \sec^2 \theta}$

$\frac{1}{\cos 2\theta} = \frac{1}{\cos^2 \theta} \cdot \frac{1}{2\cos^2 \theta - 1}$
 $\frac{1}{\cos 2\theta} = \frac{1}{\cos 2\theta}$

19. $\cos^4 x - \sin^4 x = \cos 2x$

$(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) = \cos 2x$
 $\cos 2x = \cos 2x$

20. $(\sin x + \cos x)^2 = 1 + \sin 2x$

$\sin^2 x + 2\sin x \cos x + \cos^2 x = 1 + \sin 2x$
 $1 + 2\sin x \cos x = 1 + 2\sin x \cos x$

Cumulative Review from Test 1-5A:

21. Identify the following conics: a. $\frac{(x-3)^2}{25} + \frac{y^2}{9} = 1$ *ellipse*

b. $(x+1)^2 + y^2 = 16$ *circle*

22. Multiply the following matrices: $\begin{bmatrix} x & -1 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 3x+2 & 2x-1 \\ 0 & 7 \end{bmatrix}$

23. Solve the linear system: $2x + 4y = 8$
 $x - 2y = 12$
 $x = 0$ $y = -2$

24. Find a positive co-terminal angle to: a. $\theta = -\frac{2\pi}{7}$ *or* $\frac{12\pi}{7}$ b. $\theta = \frac{\pi}{5}$ *or* $\frac{11\pi}{5}$

25. If $\tan \theta = \frac{5}{12}$ and θ is in quadrant 3, what is the exact value of $\cos \theta$? $-\frac{12}{13}$

26. Find the reference angle: a. $\theta = 120^\circ$ 60° b. $\theta = 315^\circ$ 45°

27. Find the exact value of the following function: $\sin\left(-\frac{4\pi}{3}\right) = \frac{\sqrt{3}}{2}$

28. Evaluate: $\cos^{-1}\left(\frac{1}{2}\right) = 120^\circ$