GSE PreCalculus: Unit 4 – Trig Identities
WS 5.10 – Solving Trig Equations (2)

Name _____
Date _____ Day ___

Solve the following equations.

1. $\tan\theta+\sqrt{3}=0 \qquad \theta\in[0°,360°]$

$\tan\theta=-\sqrt{3}$

$\theta=-60+360$

tan = − in Quad 2, 4

Quad 4 = 300

Quad 2 = 120

ref for 300 (outside)

$120°, 300°$

2. $2\cos\theta+\sqrt{3}=0 \qquad \theta\in[0°,360°]$

$2\cos\theta=-\sqrt{3}$

$\cos\theta=-\sqrt{3}/2 \rightarrow$ Quad 2, 3 (150, 210)

ref for 150 (outside)

$150°, 210°$

3. $2\sin(\theta+47°)=1 \qquad \theta\in[0°,360°]$

$\sin(\theta+47°)=\frac{1}{2}$

$\theta+47=\sin^{-1}(\frac{1}{2}) \rightarrow$ Quad 1, 2 (30, 150)

$\theta+47=30 \quad \theta+47=150$

$\theta=-17 \quad \theta=103$

$+343° \quad -17°, 103°$

4. $\sec(\theta+81°)=2 \qquad \theta\in[0°,360°]$

$\frac{1}{\cos}(\theta+81)=2$

$\cos(\theta+81)=\frac{1}{2}$

$\theta+81=\cos^{-1}(\frac{1}{2}) \rightarrow$ Quad 1, 4 (60, 300)

$\theta+81=60 \quad \theta+81=300$

$339° \quad -21°, 219°$

5. $4\cos^2\theta=1 \qquad \theta\in[0°,90°]$

$\cos^2\theta=1/4$

$\cos\theta=\pm\frac{1}{2} \rightarrow$ Q1

$\cos^{-1}(\frac{1}{2})$

$60°$

6. $4\cos^2\theta=3 \qquad \theta\in[90°,180°]$

$\cos^2\theta=\frac{3}{4}$

Q2

$\cos\theta=\pm\frac{\sqrt{3}}{2} \quad \cos\theta=-\frac{\sqrt{3}}{2}$

$150°$

7. $2\sin\theta\cos\theta=\sqrt{2}\cos\theta \qquad \theta\in[180°,270°]$

Q3

$2\sin\theta\cos\theta-\sqrt{2}\cos\theta=0$

$\cos\theta(2\sin\theta-\sqrt{2})=0$

$\cos\theta=0 \quad 2\sin\theta-\sqrt{2}=0$

~~90~~, 270 $\quad \sin\theta=\sqrt{2}/2$

~~45~~, ~~135~~ $\quad 270$

8. $\tan\theta\sec\theta=\tan\theta \qquad \theta\in[0°;360°]$

$\tan\theta\sec\theta-\tan\theta=0$

$\tan\theta(\sec\theta-1)=0$

$\tan\theta=0 \quad \sec\theta-1=0 \quad 0°, 360°$

0, 360 $\quad \sec\theta=1$

$\cos\theta=1$

0

9. $\cos\theta+2=3\cos\theta \qquad \theta\in[-90°,90°]$

$-\cos\theta \quad -\cos\theta$

$\cos\theta$

$\frac{2}{2}=\frac{2\cos\theta}{2}$

Q = 1, 4

$1=\cos\theta$

$\cos^{-1}(1)$

$0°$

10. $2\cos^2\theta-5\cos\theta+2=0 \qquad \theta\in[0°,360°]$

$(2\cos\theta-1)(\cos\theta-2)$

$2\cos\theta-1=0 \quad \cos\theta=2$

$\cos\theta=\frac{1}{2}$

$\cos^{-1}(\frac{1}{2})$

$60°, 300°$ $\quad 60°, 300°$

11. $2\sec^2\theta - 3\sec\theta - 2 = 0 \qquad \theta \in [0°, 90°]$

$2A^2 - 3A - 2$

$(2A+1)(A-2)$

$2\sec\theta + 1 = 0$

$2\sec\theta = -1$

$\sec\theta = -\frac{1}{2}$

$\frac{1}{\cos\theta} = -\frac{1}{2}$

$\cos\theta = -2$ ✗

$\sec\theta - 2 = 0$

$\sec\theta = 2$

$\frac{1}{\cos\theta} = \frac{2}{1}$

$\cos\theta = \frac{1}{2}$

$60°$

12. $\sin^2\theta + 5\sin\theta + 6 = 0 \qquad \theta \in [0°, 360°]$

$A^2 + 5A + 6$

$(A+2)(A+3)$

$\sin\theta + 2 = 0$ $\qquad \sin\theta + 3 = 0$

$\sin\theta = -2$ ✗ $\qquad \sin\theta = -3$ ✗

No solution

13. $(5\sin x - 2)(3\sin x + 2) = 0 \qquad \theta \in [0°, 360°]$

$\sin x = \frac{2}{5} \qquad \sin x = -\frac{2}{3}$

$23.58°$ $\qquad -41.81°$

$+360$

318.19

14. $\sin x = \sin 2x \qquad \theta \in [0°, 180°]$

$\sin x - 2\sin x\cos x = 0$

$\sin x(1 - 2\cos x) = 0$

$\sin x = 0 \qquad 1 - 2\cos x = 0$

$-2\cos x = -1$

$\cos x = \frac{1}{2}$

$0° \qquad 60°$

$0, 60°$

Verify the following identities:

1. $\sin x \cos x \tan x = 1 - \cos^2 x$

$\sin x \cos x \left(\frac{\sin x}{\cos x}\right) = 1 - \cos^2 x$

$\sin^2 x = 1 - \cos^2 x$

$\sin^2 x = \sin^2 x$

2. $\tan x + \cot x = \sec x \csc x$

$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$

$\frac{\sin^2 x}{\cos x \sin x} + \frac{\cos^2 x}{\cos x \sin x} = \sec x \csc x$

$\frac{1}{\cos x \sin x} = \sec x \csc x$

$\frac{1}{\cos x \sin x} = \frac{1}{\cos x \sin x}$

3. $\frac{1 + \cos x}{\sin x} = \frac{\sin x}{1 - \cos x}$

$1 - \cos^2 x = \sin^2 x$

$\sin^2 x = \sin^2 x$

4. $\sin x \sec x = \tan x$

$\sin x\left(\frac{1}{\cos x}\right) = \tan x$

$\frac{\sin x}{\cos x} = \tan x$

$\tan x = \tan x$

5. $\sin\theta(\cot\theta + \tan\theta) = \sec\theta$

$\sin\theta\left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right) = \sec\theta$

$\cos\frac{\cos\theta}{\cos} + \frac{\sin^2\theta}{\cos\theta} = \sec\theta$

$\frac{\cos^2\theta + \sin^2\theta}{\cos\theta} = \sec\theta$

$\frac{1}{\cos\theta} = \sec\theta \qquad \sec\theta = \sec\theta$

6. $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$

$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta = 2$

$1 + 1 = 2$

$2 = 2$

7. $\csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$

$(\csc^2 x + \cot^2 x)(\csc^2 x - \cot^2 x) = \csc^2 x + \cot^2 x$

(1)

$\csc^2 x + \cot^2 x = \csc^2 x + \cot^2 x$

$1 + \cot^2\theta = \csc^2\theta$

$-\cot^2\theta \qquad -\cot^2\theta$