GPS PreCalculus Unit 4: Trig Identities
Worksheet 4.2

Name _____
Date _____ **Day** ____

Verify the identity algebraically

1. $\csc\theta\tan\theta=\sec\theta$

$\left(\frac{1}{\sin\theta}\right)\left(\frac{\sin\theta}{\cos\theta}\right)=\sec\theta$

$\frac{1}{\cos\theta}=\sec\theta$

$\sec\theta=\sec\theta$

2. $\cos\theta\sec\theta-\cos^2\theta=\sin^2\theta$

$\cos\theta\left(\frac{1}{\cos\theta}\right)-\cos^2\theta=\sin^2\theta$

$1-\cos^2\theta=\sin^2\theta$

$\sin^2\theta=\sin^2\theta$

3. $\frac{\csc\theta}{\sec\theta}+\frac{\cos\theta}{\sin\theta}=2\cot\theta$

Hint: simplify $\frac{\csc\theta}{\sec\theta}$ in terms of $\sin\theta,\cos\theta$

$\frac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}}+\frac{\cos\theta}{\sin\theta}=2\cot\theta$

$\frac{1}{\sin\theta}\cdot\frac{\cos\theta}{1}+\frac{\cos\theta}{\sin\theta}=2\cot\theta$

$\frac{\cos\theta}{\sin\theta}+\frac{\cos\theta}{\sin\theta}=2\cot\theta$

$2\cot\theta=2\cot\theta$

4. $\frac{\sec^2\theta-\tan^2\theta+\tan\theta}{\sec\theta}=\cos\theta+\sin\theta$ Very Challenging!!!

$\frac{(1+\tan^2\theta)-\tan^2\theta+\tan\theta}{\sec\theta}=\cos\theta+\sin\theta$

$\frac{1+\tan\theta}{\sec\theta}$

$\frac{\frac{\cos}{\cos}(1+\frac{\sin\theta}{\cos\theta})}{\frac{1}{\cos\theta}}=\frac{\cos\theta+\sin\theta}{\cos\theta}\cdot\frac{\cos\theta}{1}=\cos\theta+\sin\theta$

$\cos\theta+\sin\theta=\cos\theta+\sin\theta$

5. $\sin t\csc t=1$

$\frac{\sin t}{1}\left(\frac{1}{\sin t}\right)=1$

$1=1$

6. $\tan y\cot y=1$

$\frac{\sin y}{\cos y}\left(\frac{\cos y}{\sin y}\right)=1$

$1=1$

7. $\frac{\csc^2 x}{\cot x}=\csc x\sec x$

$\frac{1}{\sin^2 x}\cdot\frac{\sin x}{\cos x}=\csc x\sec x$

$\frac{1}{\sin x}\cdot\frac{1}{\cos x}=\csc x\sec x$

$\csc x\sec x=\csc x\sec x$

8. $\cot^2 y(\sec^2 y-1)=1$ Hint: rewrite $\sec^2 y-1$

$\tan^2 y+1=\sec^2 y$

$\cot^2 y(\tan^2 y)=1$

$\left(\frac{\cos^2 y}{\sin^2 y}\right)\left(\frac{\sin^2 y}{\cos^2 y}\right)=1$

$1=1$

9. $\cos^2\beta-\sin^2\beta=2\cos^2\beta-1$

$\cos^2\beta-(1-\cos^2\beta)=2\cos^2\beta-1$

$\cos^2\beta-1+\cos^2\beta=2\cos^2\beta-1$

$2\cos^2\beta-1=2\cos^2\beta-1$

10. $\tan^2\theta+6=\sec^2\theta+5$

$(\sec^2\theta-1)+6=\sec^2\theta+5$

$\sec^2\theta+5=\sec^2\theta+5$

$\tan^2+1=\sec^2\theta-1$

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11. $2-\csc^2 z = 1-\cot^2 z$

~~Hint: rewrite $\tan^2 z$ as 1~~

$2-(1+\cot^2 z) = 1-\cot^2 z$

$2-1-\cot^2 z = 1-\cot^2 z$

$1-\cot^2 z = 1-\cot^2 z$

12. $\cos x + \sin x \tan x = \sec x$

~~Hint: rewrite $\csc^2 z$~~

$\cos x + \sin x\left(\frac{\sin x}{\cos x}\right) = \sec x$

$\cos x \left(\frac{\cos x}{\cos x} + \frac{\sin^2 x}{\cos x}\right) = \sec x$

$\frac{\cos^2 x + \sin^2 x}{\cos x} = \sec x$

$\frac{1}{\cos x} = \frac{1}{\cos x}$

13. ~~$\frac{\cot^3 t}{\csc t} = \cos t(\csc^2 t - 1)$~~ Bad problem

$\frac{\cot^2 t(\cot t)}{\csc t} = \cos t(\csc t - 1)$

$\frac{(\csc^2 - 1)\cot t}{\csc t} = \cos t(\csc t - 1)$

14. $\sec^4\theta - \tan^4\theta = 1 + 2\tan^2\theta$

Hint: factor as a difference of two squares

$x^4 - y^4 = (x^2 - y^2)(x^2 + y^2)$

$(\sec^2\theta - \tan^2\theta)(\sec^2\theta + \tan^2\theta) = 1 + 2\tan^2\theta$

$(1 + \tan^2\theta - \tan^2\theta)(1 + \tan^2\theta + \tan^2\theta) = 1 + 2\tan^2\theta$

$1(1 + 2\tan^2\theta) = 1 + 2\tan^2\theta$

$1 + 2\tan^2\theta = 1 + 2\tan^2\theta$

15. $\csc^4\theta - \cot^4\theta = 2\csc^2\theta - 1$

Hint: factor as a difference of two squares

$x^4 - y^4 = (x^2 - y^2)(x^2 + y^2)$

$(\csc^2\theta - \cot^2\theta)(\csc^2\theta + \cot^2\theta) = 2\csc^2\theta - 1$

$(1 + \cot^2\theta - \cot^2\theta)(\csc^2\theta + \csc^2\theta - 1) = 2\csc^2\theta - 1$

$1(2\csc^2\theta - 1) = 2\csc^2\theta - 1$

16. $\frac{\csc\theta}{\sin\theta} - \frac{\cot\theta}{\tan\theta} = 1$

$\frac{1}{\sin\theta}\cdot\frac{1}{\sin\theta} - \frac{\cos}{\sin\theta}\cdot\frac{\cos}{\sin\theta} = 1$

$\frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta} = 1$

$\frac{1-\cos^2\theta}{\sin^2\theta} = 1$

$\frac{\sin^2\theta}{\sin^2\theta} = 1$

$1 = 1$

17. $\frac{1^{(1-\cos\theta)}}{1+\cos\theta} + \frac{1^{(1+\cos\theta)}}{1-\cos\theta} = 2\csc^2\theta$

$\frac{(1-\cos\theta)+(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)} = 2\csc^2\theta$

$\frac{2}{(1-\cos^2\theta)} = 2\csc^2\theta$

$\frac{2}{\sin^2\theta} = 2\csc^2\theta$

$2\left(\frac{1}{\sin^2\theta}\right) = 2\csc^2\theta$

$2\csc^2\theta = 2\csc^2\theta$