

5.9

Verify that the x-values are solutions of the equation.

1. $2 \cos x - 1 = 0$ (a) $x = \frac{\pi}{3}$ *yes* (b) $x = \frac{5\pi}{3}$ *yes*

$2 \cos(\frac{\pi}{3}) - 1 = 0$ $2 \cos x = 1$
 $\cos x = \frac{1}{2}$

2. $2 \cos^2 x + 3 \cos x + 1 = 0$ (a) $x = \frac{4\pi}{3}$ *yes* (b) $x = \pi$ *yes*

$2 \cos(\frac{4\pi}{3})^2 + 3 \cos(\frac{4\pi}{3}) + 1 = 0$

Solve the equations for $0 \leq x < 2\pi$

3. $2 \cos x + 1 = 0$
 $2 \cos x = -1$
 $\cos x = -\frac{1}{2}$
 $120^\circ, 240^\circ$

4. $\sqrt{3} \sec x - 2 = 0$
 $\sec x = \frac{2}{\sqrt{3}}$
 $\cos x = \frac{\sqrt{3}}{2}$
 $30^\circ, 330^\circ$

$\frac{\sqrt{3} \sec x = 2}{\sqrt{3} \quad \sqrt{3}}$
 $\sec x = \frac{2}{\sqrt{3}}$

5. $3 \csc^2 x - 4 = 0$
 $\sqrt{\csc^2 x} = \sqrt{\frac{4}{3}}$
 $\csc x = \pm \frac{2}{\sqrt{3}}$
 $\sin x = \pm \frac{\sqrt{3}}{2}$
 $\pm 60^\circ$ OR $60^\circ, 300^\circ$
 $120^\circ, 240^\circ$

6. $2 \sin^2 2x = 1$
 $\sqrt{\sin^2 2x} = \sqrt{\frac{1}{2}}$
 $\sin 2x = \pm \sqrt{\frac{1}{2}} = \pm \frac{1}{\sqrt{2}}$
 $\frac{2x}{2} = \frac{45^\circ}{2}$ $\frac{2x}{2} = -\frac{45^\circ}{2}$
 $\pm 22.5^\circ$ 22.5°
 $+ 67.5^\circ$ $+ 137.5^\circ$

7. $\csc^2 x - 2 = 0$
 $\sqrt{\csc^2 x} = \sqrt{2}$
 $\csc x = \pm \sqrt{2}$
 $\sin x = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$
 $\pm 45^\circ$ OR $45^\circ, 315^\circ$
 $135^\circ, 225^\circ$

8. $4 \cos^2 x - 3 = 0$
 $\sqrt{\cos^2 x} = \sqrt{\frac{3}{4}}$
 $\cos x = \pm \frac{\sqrt{3}}{2}$
 $30^\circ, 150^\circ$
 $210^\circ, 330^\circ$

9. $\sin^2 x = 3\cos^2 x$

$\sin^2 x - 3\cos^2 x = 0$

$\sin^2 x - 3(1 - \sin^2 x) = 0$

$\sin^2 x - 3 + 3\sin^2 x = 0$

$4\sin^2 x - 3 = 0$

$\sqrt{\sin^2 x} = \sqrt{\frac{3}{4}}$

$\sin x = \pm \frac{\sqrt{3}}{2}$

$\pm 60^\circ$
OR

$60^\circ, 300^\circ$

$120^\circ, 240^\circ$

10. $(3\tan^2 x - 1)(\tan^2 x - 3) = 0$

$\tan^2 x = \frac{1}{3}$

$\tan^2 x = 3$

$\tan x = \pm \frac{1}{\sqrt{3}}$

$\tan x = \pm \sqrt{3}$

$\pm 30^\circ$
 $30^\circ, 330^\circ$

$\pm 60^\circ$
 $60^\circ, 300^\circ$

11. $\tan^2 x - 1 = 0$

$\tan^2 x = 1$

$\tan x = \pm 1$

$\pm 45^\circ$ OR $45^\circ, 315^\circ$
 ~~$135^\circ, 225^\circ$~~

12. $3\tan^3 x = \tan x$

$3\tan^3 x - \tan x = 0$

$\tan x (3\tan^2 x - 1) = 0$

$\tan x = 0$

$3\tan^2 x - 1 = 0$

$\sqrt{\tan^2 x} = \sqrt{\frac{1}{3}}$

$\tan x = \pm \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

~~$0^\circ, \pm 30^\circ$~~
OR
 ~~$0^\circ, 30^\circ, 330^\circ$~~

$0^\circ, 180^\circ, 30^\circ, 150^\circ, 210^\circ, 330^\circ$

13. $2\sin^2 x = 2 + \cos x$

$2\sin^2 x - \cos x - 2 = 0$

$2(1 - \cos^2 x) - \cos x - 2 = 0$

~~$2\cos^2 x - \cos x - 2 = 0$~~

$-2\cos^2 x - \cos x = 0$

$\cos x (-2\cos x - 1) = 0$

$\cos x = 0$ $-2\cos x - 1 = 0$

$\cos x = -\frac{1}{2}$

$270^\circ, 240^\circ, 90^\circ, 120^\circ$

15. $\sec x \csc x = 2 \csc x$

$\sec x \csc x - 2 \csc x = 0$

$\csc x (\sec x - 2) = 0$

$\csc x = 0$ / $\sec x = 2$

$\sin x = 1/0$ $\cos x = 1/2$

$60^\circ, 300^\circ$

17. $2\sec^2 x + \tan^2 x - 3 = 0$

$2(1 + \tan^2 x) + \tan^2 x - 3 = 0$

$2 + 2\tan^2 x + \tan^2 x - 3 = 0$

$3\tan^2 x - 1 = 0$

$\sqrt{\tan^2 x} = \sqrt{\frac{1}{3}}$

$\tan x = \pm \frac{1}{\sqrt{3}}$

$\pm 30^\circ$
OR
 $30^\circ, 330^\circ$
 $150^\circ, 210^\circ$

14. $\sec^2 x - \sec x = 2$

$\sec^2 x - \sec x - 2 = 0$

$(\sec x - 2)(\sec x + 1) = 0$

$\sec x = 2$ $\sec x = -1$

$\cos x = 1/2$ $\cos x = -1$

$60^\circ, 300^\circ, 180^\circ$

16. $2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 1)(\sin x + 1) = 0$

$\sin x = -\frac{1}{2}$ $\sin x = -1$

$-30^\circ, -90^\circ$
OR
 $210^\circ, 330^\circ$ OR 270°