Two ways to find the Area of a triangle:

1. $A_\triangle = \dfrac{1}{2}ab\sin C$ *Given 2 sides and 1 angle – all letters different; a, b = sides, C = angle*

Example: Find the area: p = 6.8 in, k = 16 in, H = $111°$

$$A = \frac{1}{2}kp\sin H$$
$$A = \frac{1}{2}(6.8)(16)\sin 111°$$
$$A = 50.79 in^2$$

2. *Heron's Area Formula - Given 3 sides of the triangle*

The area of a triangle with sides of length *a*, *b*, and *c* is

$$A_\triangle = \sqrt{s(s-a)(s-b)(s-c)}$$

Where $s = \dfrac{1}{2}(a+b+c)$. The variable *s* is called the *semiperimeter*, or half-perimeter, of the triangle

Example: Find the area of the triangle to the right (nearest tenth).
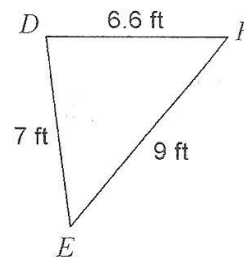
Step 1: Find the semiperimeter:
$$s = \frac{1}{2}(7+6.6+9) = 11.3$$

Step 2: Use Heron's formula:
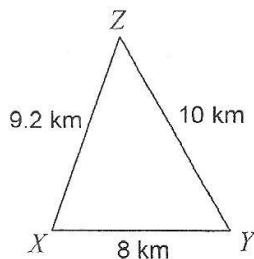$$A_\triangle = \sqrt{11.3(11.3-7)(11.3-6.6)(11.3-9)}$$
$$A_\triangle = \sqrt{525.2579}$$
$$A_\triangle \approx 22.9$$

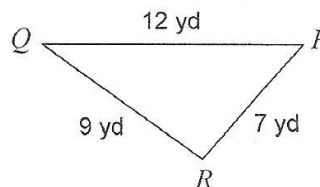*Exercises:* Find the area of the triangle to the nearest tenth (show work).

1.

S=13.6

34.7 km²

$A = \sqrt{13.6(13.6-9.2)(13.6-10)(13.6-8)}$

2.

S=14

31.3 yd²

$A = \sqrt{14(14-9)(14-12)(14-7)}$

3. In $\triangle RPQ$, $q = 14$, $p = 12$, $r = 9$ $s=17.5$

$$53.5 \ u^2$$

$$A = \sqrt{17.5(17.5-14)(17.5-12)(17.5-9)}$$

4. In $\triangle XYZ$, $y = 13.9$, $x = 15.6$, $z = 8.1$ 18.8

$$56.2 \ u^2$$

$$A\sqrt{18.8(18.8-13.9)(18.8-15.6)(18.8-8.1)}$$

5. In $\triangle ZXY$, $y = 8.6$, $x = 6$, $m\angle Z = 84°$

$$25.7 \ u^2$$

$$A = \frac{1}{2}(8.6)(6)\sin 84$$

6. In $\triangle TRS$, $s = 8$, $r = 17$, $m\angle T = 82°$

$$67.3 \ u^2$$

$$A = \frac{1}{2}(8)(17)\sin 82°$$

7. In $\triangle RPQ$, $r = 11$, $q = 10.1$, $p = 8$ 14.55

$$39.2 \ u^2$$

$$A = \sqrt{14.55(14.55-11)(14.55-10.1)(14.55-8)}$$

Review of Law of Sines/Law of Cosines

8. *Solve* the triangle:

In $\triangle PQR$, r=13m, q = 6m, $m\angle P=96°$

$p=14.9$ $P=96$
$r=13$ $R=60.2$
$q=6$ $Q=23.8$

$p^2 = 13^2 + 6^2 - 2(13)(6)\cos 96$
$p=14.9$

$\dfrac{\sin 96}{14.9} = \dfrac{\sin R}{13}$
$60.2 = R$

9. *Solve* the triangle:

In $\triangle PQR$, q=22 ft, r = 6 ft, p = 20ft

$p=20$ $P=62.9$
$q=22$ $Q=101.5$
$r=6$ $R=15.6$

$22^2 = 20^2 + 6^2 - 2(20)(6)\cos Q$
$484 = 436 - 240\cos Q$
$101.5 = Q$

$\dfrac{\sin 101.5}{22} = \dfrac{\sin P}{20}$

10. *Solve* the triangle:

In $\triangle BCA$, $m\angle B = 26°$, a = 14cm, b = 10cm

$A=37.9$ $a=14$
$B=26$ $b=10$
$C=116.1$ $c=20.5$

$\dfrac{\sin 26}{10} = \dfrac{\sin A}{14}$
$A = 37.9$

$\dfrac{\sin 116.1}{c} = \dfrac{\sin 26}{10}$

$A=142.1$ $a=14$
$B=26$ $b=10$
$C=11.9$ $c=4.7$

$\dfrac{\sin 26}{10} = \dfrac{\sin 11.9}{c}$

11. *Solve* the triangle:

In $\triangle BCA$, $m\angle B = 129°$, a = 37in, b = 37in

No solution